

13. (2)

14. (2)

15. (2)

16. (2)

17. (1)

18. (1)

$$\lim_{x \rightarrow 0^+} = f(0) = \lim_{x \rightarrow 0} \frac{2 \sin^2 \frac{x}{2}}{x} = k$$

19. (1)

20. (3)

$$\left(\tan^{-1} x + \cot^{-1} x \right) + \cot^{-1} x = \frac{2\pi}{3}, \cot^{-1} x = \frac{\pi}{6}$$

21. (2)

Remember that $\int \sqrt{x^2 + 1} dx$

$$= \frac{x}{2} \sqrt{x^2 + 1} + \frac{1}{2} \log \left(x + \sqrt{x^2 + 1} \right)$$

$$\therefore \frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 + 1} + \frac{1}{2} \log \left(x + \sqrt{x^2 + 1} \right) \right)$$

$$= \sqrt{x^2 + 1}$$

22. (2) $x^2 + y^2 = 25$ (1)

$$2x + 2y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-x}{y}$$

Slope of the line $3x - 4y = 7$ is $m = 3/4$

Since tangent is parallel to the given line

$$\frac{dy}{dx} = \frac{3}{4} \Rightarrow -\frac{x}{y} = \frac{3}{4} \Rightarrow y = -\frac{4x}{3} \dots(2)$$

$$\text{From eq.(1)} \quad x^2 + \frac{16}{9}x^2 = 25 \Rightarrow x = \pm 3$$

$$\text{If } x = 3, \text{ from eq.(ii) } y = -\frac{4}{3}(3) = -4$$

$$\text{If } x = -3 \text{ from eq. (2) } y = -\frac{4}{3}(-3) = 4$$

 \therefore The points are (3,-4) and (-3,4)23. (4) $y^2 = 2ax$ (1)

$$\text{Put } x = \frac{a}{2}, y^2 = 2a \frac{a}{2}, y = \pm a,$$

 \therefore The points are $\left(\frac{a}{2}, a\right)$ and $\left(\frac{a}{2}, -a\right)$

on differentiation of (1) w.r.t x we get

$$2y \frac{dy}{dx} = 2a \Rightarrow \frac{dy}{dx} = \frac{a}{y}$$

$$\text{At } \left(\frac{a}{2}, a\right) \frac{dy}{dx} = \frac{a}{y} = \frac{a}{a} = 1 = m_1$$

$$\text{At } \left(\frac{a}{2}, -a\right) \frac{dy}{dx} = \frac{a}{y} = \frac{a}{-a} = -1 = m_2$$

Since $m_1 \times m_2 = -1$, the two tangents are at right angles.

24. (1)

$$f'(x) = \frac{2}{3}(x-1)^{-1/3} \text{ which does not exist for}$$

 $x = 1$. \therefore $f(x)$ is not derivable in any interval containing 1. \therefore $f'(x)$ exists is (1,2) \therefore Mean value theorem is applicable in (1,2)

25. (2)

$$`*` \text{ is commutative as } b * a = |b - a| - 1$$

$$= |a - b| - 1 = a * b$$

$$(\because -x = |x| \forall x \in \mathbf{R})$$