

1. (1)

$$E = \frac{-dV}{dr}$$

$$E = \frac{\partial V}{\partial r} = \left[-\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k} \right]$$

$$= \left[\hat{i}(2xy + z^3) + \hat{j}x^2 + \hat{k}3xz^2 \right]$$

2. (1)

$$C_{\text{middle}} = 1.5 + 1.5 = 3\mu\text{F}$$

Then 3 are in series

3. (4) $P = P_1 + P_2$

4. (1) Balancing length is not affected

5. (1), Resistance required in series is

$$R = \frac{V}{I_g} - G = \frac{nV}{\left[\frac{V}{G}\right]} - G = (n-1)G$$

6. (1)

$$\text{Magnetisation} = \frac{\text{dipole moment}}{\text{volume}}$$

$$= \frac{8 \times 10^{10} \times 9 \times 10^{-24}}{10^{-18}}$$

$$= 7.2 \times 10^5 \text{ Am}^{-1}$$

7. (2)

$$V_L = V_C = 300 \text{ V}$$

circuit is under resonance

$$\therefore V_R = V = 220 \text{ V}$$

$$Z = R = 100 \Omega$$

$$I = \frac{V}{Z} = \frac{220 \text{ V}}{100 \Omega} = 2.2 \text{ A}$$

8. (4)

9. (4)

10. (3)

11. (3)

12. (2)

13. (1)

$$\log K = 0.3010 \quad K = 2$$

$$\frac{1}{n} = \tan 45^\circ, n = 1$$

$$\frac{x}{m} = Kp^{1/n} = 2 \times (0.6)^1 = 1.2$$

14. (2)

$$\frac{-Ea}{RT} = \frac{-2800K}{T} \quad Ea = 232.79 \text{ kJmol}^{-1}$$

15. (1)

16. (2)

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6} \quad \text{--- (1)}$$

$$\sin^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad \text{--- (2)}$$

Solving (1) & (2) $x = \frac{\sqrt{3}}{2}$ is the unique solution.

17. (1)

$$f(x) = \cos^{-1} \left(\frac{x^2-1}{x^2+1} \right) = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = \pi - \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

put $x = \tan \theta$. Then $f(x) = \pi - 2\theta = \pi - 2 \tan^{-1} x$

$$f'(x) = \frac{-2}{1+x^2}$$

18. (1)

19. (2)

20. (4) If e is the identity element, then

$$a * e = \frac{ae}{2} \Rightarrow a = \frac{ae}{2} \Rightarrow e = 2$$

21. (4)

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$ and expand, we get $x^3 = 64$

22. (2) put $x = \cos x$.

$$\text{Then } y = \sin^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\sqrt{2}} \right)$$

$$= \sin^{-1} \left[\sin \frac{\pi}{4} \cdot \cos \frac{\theta}{2} + \cos \frac{\pi}{4} \cdot \sin \frac{\theta}{2} \right]$$

$$= \sin^{-1} \sin \left(\frac{\pi}{4} + \frac{\theta}{2} \right) = \frac{\pi}{4} + \frac{1}{2} \cdot \cos^{-1} x$$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{1-x^2}}$$

23. (4)

Differentiating, we get $(1+x^2) y_1 = y$. Again differentiating

24. (4) $y^2 = 2ax$... (1)

$$\text{Put } x = \frac{a}{2}, y^2 = 2a \cdot \frac{a}{2}, y = \pm a$$

\therefore The points are $\left(\frac{a}{2}, a\right)$ and $\left(\frac{a}{2}, -a\right)$

on differentiation of (1) w.r.t x we get

$$2y \frac{dy}{dx} = 2a \Rightarrow \frac{dy}{dx} = \frac{a}{y}$$

$$\text{At } \left(\frac{a}{2}, a\right) \frac{dy}{dx} = \frac{a}{y} = \frac{a}{a} = 1 = m_1$$

$$\text{At } \left(\frac{a}{2}, -a\right) \frac{dy}{dx} = \frac{a}{y} = \frac{a}{-a} = -1 = m_2$$

Since $m_1 \times m_2 = -1$, the two tangents are at right angles.

25. (1)

$$f'(x) = \frac{2}{3} (x-1)^{-1/3} \text{ which does not exist}$$

for $x = 1$. $\therefore f(x)$ is not derivable in any interval containing 1. $\therefore f'(x)$ exists is (1,2)

\therefore Mean value theorem is applicable in (1,2)